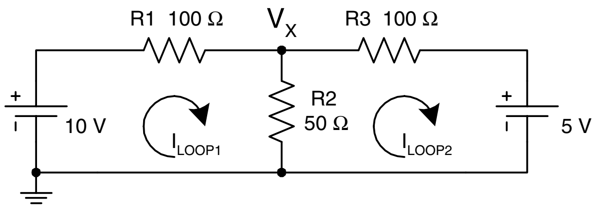

FIGURE 12.3 Double-loop circuit.

The first step is to set up the two loop equations. It is critical to adopt a consistent convention for currents and voltages. Here, positive currents flow clockwise, voltage sources are negative when they rise with the direction of the clockwise loop current, and resistors drop voltage with the same polarity as the loop current. The resulting loop equations are as follows:

$$I_{LOOP1}R1 + (I_{LOOP1} - I_{LOOP2})R2 - 10\text{ V} = 0$$

$$I_{LOOP2}R3 + (I_{LOOP2} - I_{LOOP1})R2 + 5\text{ V} = 0$$

Notice that the 10-V source appears as a negative in the first loop equation, because it represents a voltage rise, but the 5-V source appears as a positive in the second equation, because it represents a voltage drop. Our two equations with two unknowns can be solved. I_{LOOP1} can be solved as a function of I_{LOOP2} using the second equation as follows:

$$I_{LOOP1} = \frac{1}{10} + 3I_{LOOP2}$$

I_{LOOP2} is solved next by substituting the I_{LOOP1} expression into the first equation,

$$100I_{LOOP1} + 50I_{LOOP1} - 50I_{LOOP2} - 10\text{ V} = 0$$

$$150I_{LOOP1} - 50I_{LOOP2} = 10\text{ V}$$

$$\frac{150}{10} + 450I_{LOOP2} - 50I_{LOOP2} = 10\text{ V}$$

$$15 + 400I_{LOOP2} = 10\text{ V}$$

$$I_{LOOP2} = \frac{-5}{400} = -0.0125\text{ A}$$

Finally, I_{LOOP1} is solved,

$$I_{LOOP1} = 0.0625\text{ A}$$

Notice that I_{LOOP2} is a negative quantity. This is because the actual direction of current flow is out of the 5-V source rather than into it—the current is flowing counterclockwise, which is a negative clockwise current. Our results can be verified to make sure that everything adds up correctly. The voltage drops across R1 and R3 are 6.25 V and 1.25 V, respectively. Therefore, $V_X = 10\text{ V} - 6.25\text{ V} = 5\text{ V} - 1.25\text{ V} = 3.75\text{ V}$. This means that the current through R3 must be $3.75\text{ V} \div 50\ \Omega = 0.075\text{ A}$, which is exactly equal to the difference of the two loop currents ($I_{R2} = I_{LOOP1} - I_{LOOP2}$).

Node analysis is the complement to loop analysis. Rather than dealing with a fixed current around a loop in which the voltages sum to zero, node analysis examines an individual node with a fixed voltage where the currents sum to zero. Together, these loop and node analysis methods form the basis of circuit analysis. You may find that some circuits are more easily solved with one or the other. As before, the consistent usage of a polarity convention is critical to finding the correct answer. The circuit in Fig. 12.3 can be evaluated with just one node equation to determine V_X. In the following equation, the convention is taken that positive currents flow out of the node.

$$0 = \sum I_N = \frac{V_X - 10\text{ V}}{R1} + \frac{V_X - 0\text{ V}}{R2} + \frac{V_X - 5\text{ V}}{R3}$$

This node equation does not worry about whether V_X is actually higher or lower than the voltage at the other end of each resistor. If current is actually flowing into the node because the reverse voltage relationship is true, that current will have a negative polarity. Working through the algebra shows that $V_X = 3.75$ V, the same answer that was obtained using loop analysis, although with only a single equation and one unknown. If the current through any resistor must be calculated, it can be done once V_X is known.

12.3 RESISTANCE COMBINATION

Circuits can be simplified for analysis purposes when multiple resistances are present in various series and parallel topologies. Multiple resistors can be combined in arbitrary configurations and represented as a single resistance. Two resistors placed in series add, while resistors placed in parallel result in a smaller overall resistance. Resistors in parallel create a combined resistance that is less than the smallest valued resistor in the parallel group. This resulting value is determined using the inverse relationship,

$$R_{TOTAL} = \frac{1}{\sum \frac{1}{R_N}}$$

Multiple resistors placed in parallel can be indicated using a parallel bar notation. $R1 \parallel R2$ indicates that two resistors, referred to as R1 and R2, are in parallel. When two resistors are placed in a parallel arrangement, the above expression can be rewritten for this special circumstance:

$$R_{TOTAL} = \frac{R1 \times R2}{R1 + R2}$$

Figure 12.4 shows resistors in both series and parallel topologies. Series resistors R1 and R2 add to form a 150-Ω resistance. Parallel resistors R3 and R4 combine to form a smaller 33.3-Ω resistance. Placing two resistors back to back increases the total resistance observed by a current. Placing two resistors in parallel provides a second path for current to flow through, thereby reducing the overall resistance. After performing these two combinations, the circuit can be simplified a third time by adding the resulting series resistances, 150 Ω + 33.3 Ω = 183.3 Ω. When the circuit has been